TRINITY COLLEGE

Year 12 Mathematics Applications
Test 1 2017

Calculator Assumed
Categorical & Numerical Data and Recursion

STUDENT'S NAME SOLNS

DATE: Thursday 2nd March **TIME:** 50 minutes **MARKS:** 51

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (4 marks)

For each of the following sequences determine, with an appropriate calculation, whether the sequence is arithmetic, geometric or neither of these two types.

(a) 7, -14, 21, -28, 35 … [2]

$\frac{-14}{7} \neq \frac{21}{-14}$ ∴ not GP ✓

$21 - -14 \neq -14 - 7$ ∴ not AP ∴ neither ✓

(b) $\frac{10}{3}, \frac{10}{9}, \frac{10}{27}, \frac{10}{81}, \ldots$ [2]

must have calculation

$\frac{10}{9} \div \frac{10}{3} = \frac{10}{27} \div \frac{10}{9} = \frac{1}{3}$ ✓

$r = \frac{1}{3}$, ∴ sequence in a GP ✓

2. (7 marks)

John's vintage car is valued each year for insurance purposes. The value of John's car at the end of the first year is \$22 000. It was valued at \$23 650 and \$25423.75 at the end of the second and third years respectively.

(a) Show that the car values form a geometric sequence. [2]

$$\frac{25423.75}{23\,650} = \frac{23\,650}{22\,000} = 1.075$$

(b) Assuming that the value of the car continues to increase in this way,

(i) Determine the increase in the car value from the end of the first year until the end of the fourth year. [2]

$T_1 = 22\,000$

$T_4 = 27\,330.53$

$T_4 - T_1 = \$5\,330.53$

(ii) Determine the value of the car at the end of the 7th year. [1]

$T_7 = \$33\,952.63$

(iii) For how many years does John need to own the car for it to double in value? [2]

$T_n > 44\,000$ when $n = 11$

$T_{10} = 42\,179.25$

$T_{11} = 45\,342.69$

10 years after T_1 or 11 years.

3. (5 marks)

A sequence has the recursive rule $T_n = aT_{n-1} + 3$ with $T_1 = 2$ and $T_2 = 25$.

(a) Determine the value of a. [3]

$25 = 2a + 3$

$22 = 2a$

$a = 11$

(b) Determine T_4 [2]

$T_4 = 3061$

4. (3 marks)

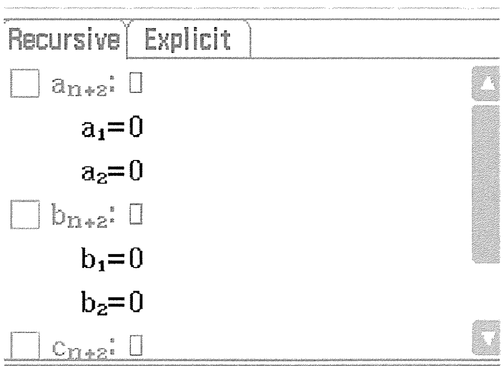
A sequence has the recursive form $T_n = 3T_{n-1} - T_{n-2} + n$ where $T_1 = 7$ and $T_2 = 12$.
Rewrite the sequence in the form A_{n+2} so that is can be entered into the classpad.

$T_3 = 3T_2 - T_1 + 3 \leftarrow n=3$

when $n=1$

$A_{n+2} = 3 \cdot A_{n+1} - A_n + n+2.$ ✓ ✓ ✓

Where $A_1 = 7$ & $A_2 = 12$. ✓



-1 per mistake.

5. (7 marks)

A survey was conducted to investigate whether the frequency with which an adult engaged in sport or exercise was ass associated with gender. A number of people were asked how many time they engaged in sport or exercise in the course of an average week. The responses are recorded below.

	Never	Once per week	Twice per week	Three times per week	Four times per week	Five times per week	
Male	36	15	18	27	30	24	150
Female	52	18	31	33	46	40	220

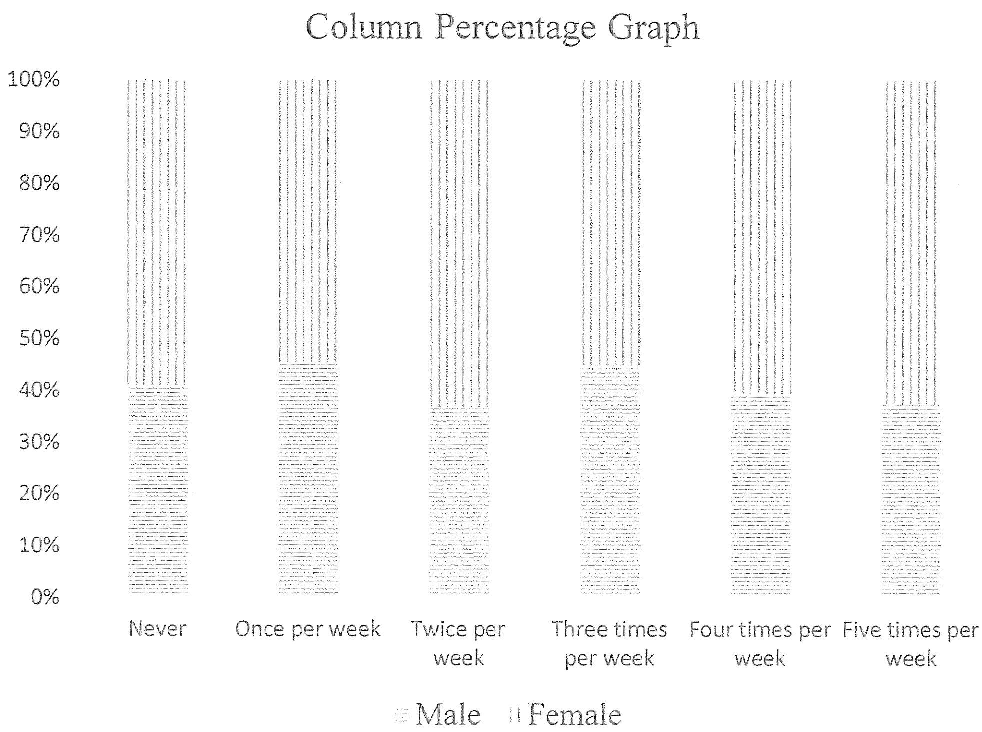
(a) State the explanatory variable and response variable. [2]

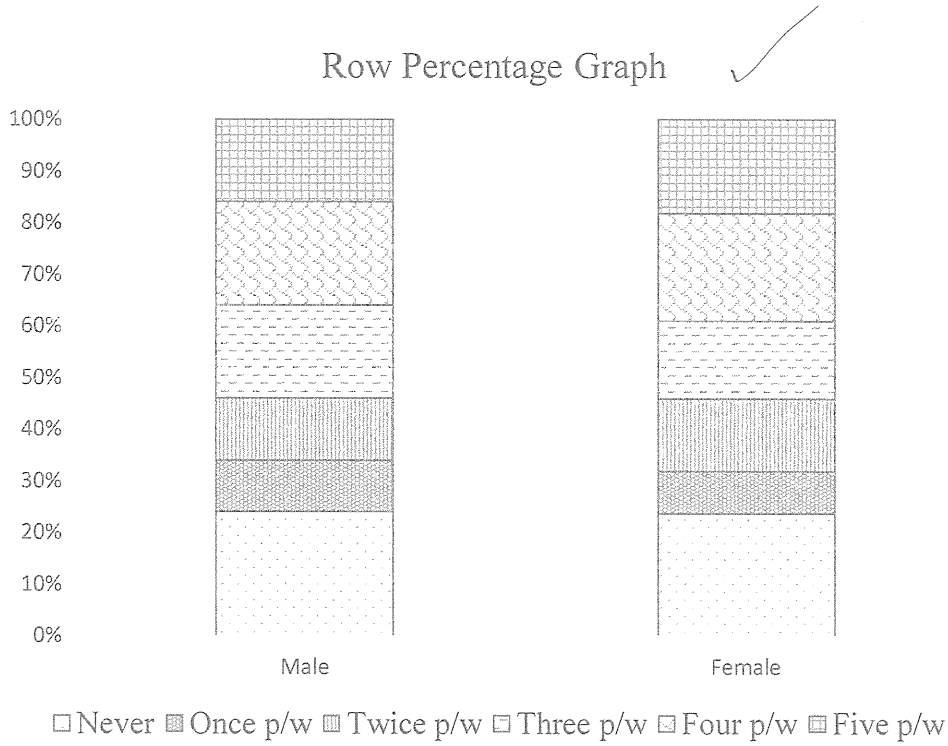
Exp : Gender

Response : Exercise Frequency.

(b) Convert the table above into a column or row percentage table as appropriate. [2]

	Never	Once per week	Twice per week	Three times per week	Four times per week	Five times per week	
Male	24%	10%	12%	18%	20%	16%	100%
Female	24%	8%	14%	15%	21%	18%	100%



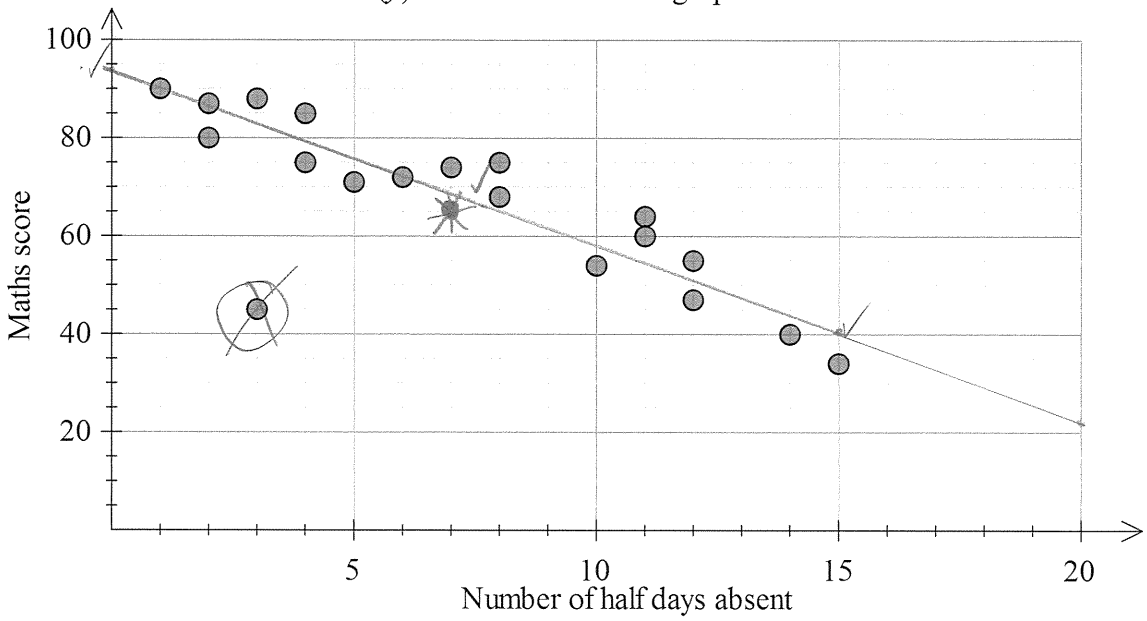


(e) Determine, with reference to the appropriate graph, whether or not the results suggest that the frequency with which an adult engages in sport or exercise is associated with gender. [3]

The row% graph shows there is no association between gender & exercise frequency. There is little variation in % across genders.

6. (20 marks)

In a recent survey of 19 students to determine if there is any relationship between maths results and the number of absences from class, the number of half days absent from school in 2016 (x) and the final score for maths (y) were recorded and graphed below.



(a) Highlight the outlier on the graph above. This point has been removed from further calculations. [1]

(b) Data for a 20th student was gathered. They scored 65 for maths and had 7 half days absent from school. Add this point to the graph. This point is included in further calculations. [1]

Linear regression was performed on the 19 data points (the outlier has been removed) and the result are shown below.

$$r^2 = 0.8916$$
$$\hat{y} = 94.2545 - 3.5693x$$

(c) State the correlation coefficient and comment on its value. [3]

$-\sqrt{0.8916} = -0.944$ Strong, negative correlation.

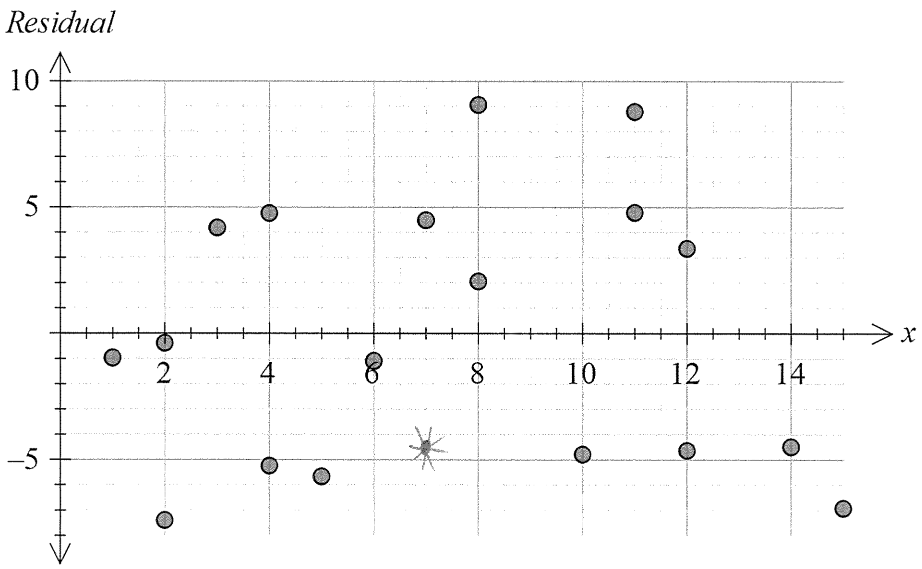
(d) State the coefficient of determination and comment on its value in the context of the question. [2]

89% of variation in maths scores can be explained by variation in absences.

$r^2 = 0.8916$

(e) Plot the line of regression on the axes above and interpret the slope in the context of the question. [3]

For every extra half day absent math scores are predicted to drop by 3.6%



(f) Add the residual for the student in part (b) to the residual plot above. [2]

$x = 7$ $\hat{y} = 69.2694$
$y = 65$
$y - \hat{y} = -4.2694$ ✓

(g) Comment on the information the residual plot reveals the researchers. [2]

The plot is random ✓ suggesting linear regression is appropriate ✓

(h) Predict the maths score for a student that had 20 half days absent in 2016 and comment on the prediction. [3]

must be rounded

$\hat{y} = 22.8685$
$\approx 23\%$ ✓

Despite strong correlation coefficient and random residual plot, it is extrapolation and should be treated with caution

(i) Given the mean number of half days absent is 7.4737 determine $\bar{y}$. [1]

$\bar{y} = 94.2545 - 3.5693(\bar{x})$
$= 67.5786$

(j) The researchers concluded that the higher maths scores are due to the low number of absences. Comment on this statement. [2]

This statement is incorrect. ✓
Cause not established. ✓

7. (5 marks)

A sequence with recursive form $P_n = b + aP_{n-1}$ has is such that $P_1 = 16, P_2 = -5$ and $P_3 = 10.75$. Determine the values of a and b and hence state the recursive rule.

$-5 = b + 16a$ ✓

$10.75 = b - 5a$ ✓

$a = -\frac{3}{4}$

$b = 7$ ✓

$\therefore \quad P_n = 7 - \frac{3P_{n-1}}{4}, \quad P_1 = 16$

✓